A Small, Fast-Response Probe to Measure Composition of a Binary Gas Mixture

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A probe to measure the concentration of the components in a binary mixture of gases is described. The probe is simple to construct and quite rugged. It samples from a very small volume, has a fast time response and can very easily detect 1% of helium in nitrogen. The explanation of the principle of operation is a good example of the power of dimensional analysis when applied to what may seem to be quite a complicated and unfamiliar problem. The analysis suggests several experiments which in turn lead to a more detailed understanding of the probe and improvements in its design.

Nomenclature

a = velocity of sound
c_p, c_v = specific heats
d = hot wire diameter
k = thermal conductivity
Kn_∞ = Knudsen number = $(\lambda_\infty/d) = (\gamma\pi/2)^{1/2}(M_\infty/Re_\infty)$
M = Mach number
MW = molecular weight
Nu_o = Nusselt number = $[q/T_w - T_r]\, d/k_o$
p = pressure
p_d = downstream pressure
q = convective heat-transfer rate
Q = additional power required to keep wire at T_w when probe is placed in a gas
R = gas constant
R_w = hot wire resistance at T_w
Re_o = Reynolds number = $\rho_\infty u_\infty d/\mu_o$
T = temperature
T_w = temperature of the wire
T_r = recovery temperature of the wire
u = velocity
U = sampled gas velocity relative to the probe
V = bridge voltage
V_o = bridge voltage for probe in vacuum
α = energy accommodation coefficient
γ = specific heat ratio = c_p/c_v
ρ = density
λ = molecular mean free path
μ = viscosity

Subscripts and Superscripts

$(\ )_o$ = stagnation conditions
$(\ )_\infty$ = freestream conditions
$(\ )^*$ = sonic conditions

Introduction

THIS work was stimulated by a need to measure the local composition in a plane turbulent mixing layer between two different gas streams. In our experiments these gases are usually nitrogen and helium. A small sampling volume, an output independent of the velocity of the fluid relative to the probe, and a response time of milliseconds or less were essential requirements to be met. The probe which was developed has some features in common with the "heat flux probe for high temperature gases" of Blackshear and Lingerson,[1] and the aspirating probe used by D'Souza, Montealegre and Weinstein.[2]

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1. Description

The probe is sketched in Fig. 1. Its construction is simple, particularly with the assistance of a glass-blower. The tip is 2 mm glass tubing drawn to a point and then polished to expose a fine hole. In our case the effective diameter of this hole is 0.001 in., determined from the measurements described in Sec. 5. Two holes approximately 0.010 in. in diam and as near as practicable to the tip were made opposite each other in the walls of the tubing with a hot tungsten wire. Bared copper leads were then glued to the outside of the tubing and an unetched Wollaston wire poked through the holes in the tubing walls and soldered at each end to the copper leads. The soldered joints and the holes were then covered with epoxy, care being taken to prevent the epoxy running along the Wollaston wire. When the glue was well cured, nitric acid was sucked into the tube and allowed to etch the wire up to the epoxy and expose the thin (0.0005 in.) platinum wire. The tubing was then slipped into a brass holder and sealed in place with shrinkable tubing.

2. Principle of Operation

The probe is attached to a vacuum pump and the platinum wire maintained at some fixed temperature T_w (i.e., resistance R_w) above its surroundings with the usual feedback bridge. If the probe is placed in a vacuum some electrical power V_o^2/R_w is required to maintain the wire at the temperature T_w because of heat conduction losses. The additional power $Q = (V^2 - V_o^2)/R_w$ required to keep the wire at this temperature when the probe is placed in a gas (or gas mixture) is then a function of the following variables

$$Q = f(p_o, \rho_o, T_o, R, C_p, \mu_o, k_o, d, T_w, p_d) \tag{1}$$

where R is the gas constant, p_d the downstream vacuum

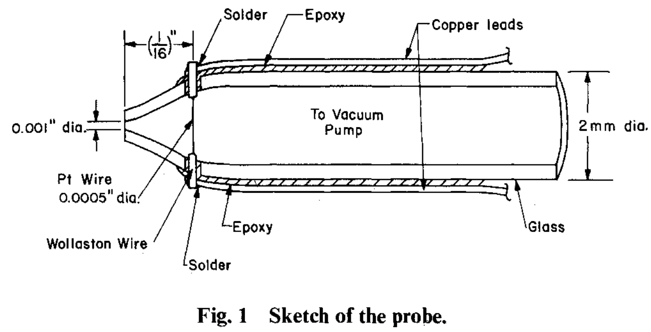

Fig. 1 Sketch of the probe.

pressure, d a characteristic dimension of geometrically similar probes (e.g., wire diameter) and the other symbols have their usual meanings. (The o subscript refers to stagnation conditions in the gas being sampled.) Dimensional analysis then requires that

$$\frac{Q}{kT_o d} = f\left(\frac{p_o}{\rho_o RT_o}, \frac{\rho_o(RT_o)^{1/2}d}{\mu_o}, \frac{\mu_o c_p}{k_o}, \frac{c_p}{R}, \frac{T_w}{T_o}, \frac{p_d}{p_o}\right) \quad (2)$$

Ideally the parameter p_d/p_o can be made arbitrarily small and negligible with a vacuum pump of sufficient capacity, in which case the output Q depends only on stagnation variables. If the sampled gas moves relative to the probe with a velocity U then, to order $(U/a_o)^2$ (a_o is the stagnation velocity of sound), all of the above parameters have the same value if evaluated at static conditions as they do at stagnation conditions. That is, for the same static temperature in the gas being sampled, the output of the probe depends on the gas and not on the velocity of the gas relative to the probe if $U \ll a_o$. The experiment described in the following sections makes it possible to state this a little more precisely. It should be noted that for perfect gases having the same Prandtl number Eq. (2) may be reduced to

$$\frac{Q}{k(T_w - T_o)d} = f\left(\gamma, \frac{\rho_o a_o d}{\mu_o}, \frac{T_w - T_o}{T_o}\right) \quad (3)$$

or, if T_o is constant

$$\frac{Q}{k(T_w - T_o)d} = f\left(\gamma, \frac{\rho_o a_o d}{\mu_o}\right) \quad (4)$$

For a given flow at the wire and small values of $(T_w - T_r)/T_r$ (where T_r is the recovery temperature of the unheated wire) one expects the equation for the additional temperature field (due to the heating of the wire) to be linear, that is for Q to be proportional to $(T_w - T_r)$. Since the recovery temperature is very nearly the stagnation temperature for circular cylinders, over a very large Reynolds number and Mach number range (Baldwin, Sandborn, Laurence[3]), T_r is approximately T_o (assuming adiabatic flow up to the wire) so that one expects Eq. (4) to apply even if there are small variations in T_o.

3. Calibration and an Experiment

The probe was placed in various gases and gas mixtures contained in a 500 cubic in. volume.

In the case of mixtures, the order in which the gases were added was varied and measurements recorded when the results were independent of this order. The volume was filled to about 105 psia and then bled slowly, measurements being made at

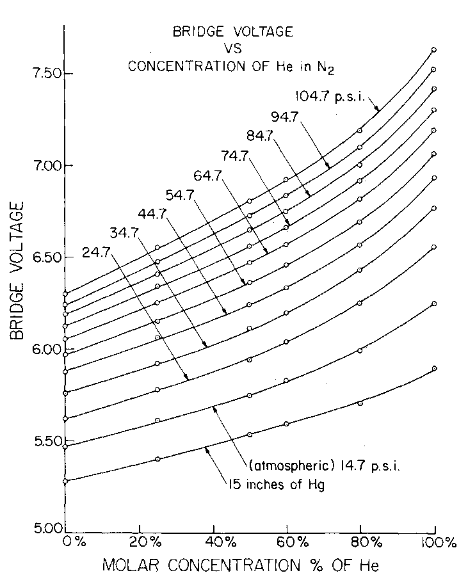

Fig. 2 Bridge voltage vs concentration of He in N_2.

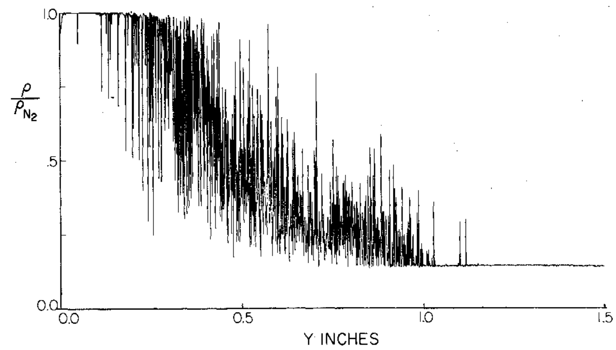

Fig. 3 Density traverse across the layer.

various pressures down to $\frac{1}{2}$ psia. During this process, the temperature of the gas mixture in the volume did not differ perceptibly from room temperature. It is worth noting that at any one pressure the voltmeter reading was steady within about 1 millivolt (cf. Fig. 2). A cross plot of bridge output in volts against molar concentration of helium in nitrogen for various pressures is shown in Fig. 2.

An example of a measurement which made use of these calibration curves is shown in Fig. 3; it is an illustration of the success with which the probe meets the requirements listed in the Introduction. The measurement consisted of traversing the probe across a plane turbulent mixing layer between nitrogen and helium (at room temperature, a pressure of 7 atm and very low Mach number) and measuring the concentration every one thousandth of an inch. The traverse was at a rate of 2 sec/in. so that a sample was obtained every 2 msec. The probe output, after A/D conversion and reduction to values of concentration is shown in Fig. 3. This computer plot draws a continuous line between individual data points, so the latter are not clearly distinguishable. Nevertheless it may be seen that the probe responds to very large changes of concentration in less than 2 msec. The ripple at each end of the traverse corresponds to changes in the least significant bit of the A/D converter. It is noted that at no point is the computed density greater than the density of N_2 or less than that of He; if there were any such points, a sensitivity to velocity would be implied. A more complete account of the experiment is given in Ref. 8.

4. Accommodation Effects

Although the calibration curves (Fig. 2) were sufficient for using the probe to measure concentration, they defied correlation in terms of Eq. (4) and we were prompted to look particularly at gases having the same γ.

Again by varying the pressure in the volume, results for He, Ar, and Kr were obtained (Fig. 4). It is clear from this figure that Eq. (4) does not correlate the measurements and it is shown in Sec. 6 that the parameter p_d/p_o, assumed insignificant, was sufficiently small for its variation to be unimportant. Evidently variables which are significant have been ignored in the dimensional analysis. Those most likely overlooked would seem to be those needed to describe accommodation effects at the wire surface, particularly the properties of the surface itself since the atomic cross section of the gas (and therefore the Knudsen number) is not an independent variable but is determined by ρ_o, a_o, and μ_o. Such effects have been observed previously with hot wires in helium (Aihara, Kassoy, Libby[4]).

If the energy accommodation coefficient at the wire surface is α, then it is expected[5] that a plot of Nu_o/α against $\rho_o a_o d/\mu_o$ should correlate the data. Values of α were chosen to give the best collapse of the data for argon and helium (shaded points in Fig. 4) onto the data for krypton. In effect, this means choosing a ratio for the accommodation coefficients of helium and krypton and a ratio for those of argon and krypton. These ratios are 0.43 for helium and 0.87 for argon. The absolute value of α for krypton is expected to be near unity. Although the accommodation coefficients of inert gases depend strongly on surface conditions[6] and surface temperature, these values for the ratios are not atypical.

5. Mass Flow in the Probe

The Reynolds number at the orifice, assuming sonic conditions, is quite large (150 to 3000) but downstream of the tip as the cross-sectional area increases it becomes correspondingly much smaller and it is perhaps not obvious that the flow is in fact choked at the tip. The following experiment answered this question and also led to conclusions about the flow conditions at the wire.

A volume was filled with gas to 105 psia and then bled through the probe orifice to approximately 40 psia. The temperature in the volume remained essentially constant. Measurements were made of the decay in gas pressure as a function of time and the results for argon and helium are plotted in Fig. 5. Evidently the rate of pressure decay is directly proportional to gas pressure (i.e., pressure is an exponential function of time) down to pressures in helium of, say, 25 psig. Above this pressure the mass flow rate (proportional to dp/dt for constant temperature in the volume) is therefore proportional to gas density and in fact the ratio of the proportionality constant for these two gases is the same as the ratio of their sound velocities. Assuming choked conditions the calculated effective orifice diameter (0.0011 in) was as near the physical diameter as we could determine with a microscope. The flow was therefore choked at the tip and the mass flow rate independent of viscosity for throat Reynolds numbers greater than, say, 300.

6. Flow Conditions at the Wire

Although it is not required for using the calibrated probe, it is of interest to try to understand flow conditions at the wire. This is not simple to determine theoretically; although viscosity has no effect on the mass flow, it will have a considerable effect on the flow up to the wire, which is in a section of the channel of much larger area. There exists the possibility of expansion to supersonic velocities, the existence of diffuser shock waves, the possibility of reaching rarefied flow conditions, etc., all of these dependent on area ratio, pressure ratio and effects of viscosity.

Knowing the mass flow, the heat-transfer rate and wire temperature, we can estimate a Nusselt number and a wire Reynolds number ($\rho ud/\mu_o$) by assuming an effective length for the wire. (It is also assumed that this length is the effective diameter of the mass flow.) Given a plot of Nu against Re for various Mach numbers (Refs. 7 and 3), an iteration leads to an estimate of the Mach number. For the probe described previously this was found to be a low value, about 0.1 with Reynolds number varying with pressure from about 1.0 to 10.

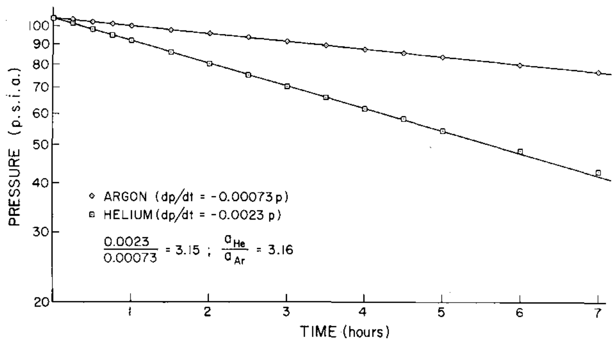

Fig. 5 Pressure vs time for Ar and He.

In this range, the slope of the Nu-Re curve on a log-log plot is closer to $\frac{1}{2}$ than 1, as indeed we observe (Fig. 4).

That the flow at the wire was evidently subsonic raised the question of whether or not the output of the wire was independent of the vacuum pump and plumbing (expressed simply by the parameter p_d/p_o). The iteration described leads to an estimate for the pressure at the wire, namely 50 mm Hg, typically. The measured pressure at the pump in this case was 15 μHg which agreed well with the manufacturer's claim of 20 μHg for the measured mass flow. Changing the pressure at the pump from 15 μHg to 3.5 mmHg produced no change in probe output. It appears then that from the tip to the pump there may be a number of sonic throats before which there is viscous compressible flow and an acceleration from subsonic to sonic velocity. This conclusion was further supported by measurements of the pressure downstream of the wire (pressure typically 1 mmHg). The ratio of this pressure to the stagnation pressure was the same for the same throat Reynolds number in helium and argon (quite different stagnation pressures), as dimensional analysis demands if p_d/p_o is negligible.

The estimated Knudsen number at the wire is less than 0.1. It is interesting that accommodation effects occur even at these low values, as has indeed been observed by other investigators.

7. Sensitivity to Velocity

To test the probe sensitivity to velocity we placed it in a uniform stream of helium at three different velocities (270 cm/sec, 1000 cm/sec and 1770 cm/sec). The output was unaffected by velocity; the relative error in bridge output voltage was smaller than $\frac{1}{2}\%$ at the highest velocity.

With the information that we have, one can estimate the error made if one determines the concentration of a moving gas with a probe that has been calibrated in stationary mixtures, the static temperatures being the same in both cases. If the Mach number of the moving gas (i.e., U/a) is M then the

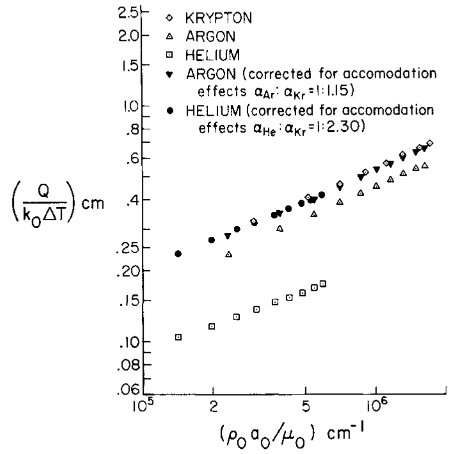

Fig. 4 $(Q/k_o\Delta T)$ **cm vs** $(\rho_o a_o/\mu_o)$ **cm**$^{-1}$**.**

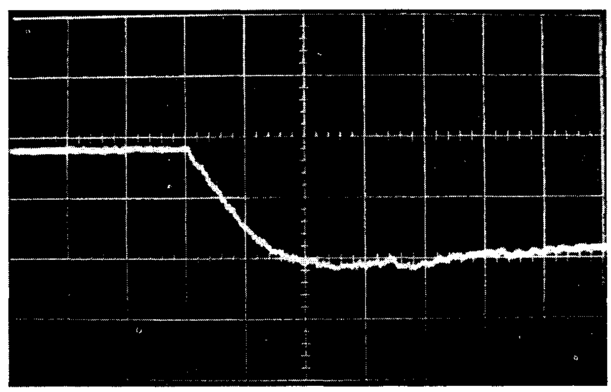

Fig. 6 Rise time output: Oscilloscope photo of response of probe to passage of shock wave; horizontal scale: 100 μ**sec/div; vertical scale: 0.05 v/div.**

relative error in the determined apparent molecular weight MW is, roughly, for small Mach number

$$\Delta(MW)/MW \simeq [T_o/(T_w - T_o) - 2]M^2 \quad (5)$$

or less, if γ and α vary with concentration.

8. Time Response

As the distance from the orifice in the tip to the wire is small and the gas velocity is of the order of the speed of sound a time response of μsec might be expected, unless the size of the hot wire and the electronics limit it to a longer time.

A new probe with a smaller wire diameter (0.0001 in.) and a less rapid area expansion (based on the findings in Sec. 6) was constructed and placed in the end wall of a shock tube. The gas in the tube was nitrogen, initially at atmospheric pressure. A shock wave passing by the probe produced an instantaneous change in the stagnation conditions of the sampled gas and the corresponding change in probe output was photographed (Fig. 6), (time scale = 100 μsec/cm, vertical scale = 0.05 v/cm). The response time is evidently about 200 μsec. The experiment was repeated using helium instead of nitrogen and, as expected, the rise time was faster. It is noted that a much longer time response is associated with the warming up of the glass.

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Supersonic Interaction in the Corner of Intersecting Wedges at High Reynolds Numbers

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The flow structure was determined in the streamwise corner formed by two intersecting wedges of 9½° half-angle in a supersonic stream at a Mach number of 3, and over a Reynolds number range from 0.4 to 60 × 10⁶. Measurements included Pitot traverses, surface pressures, and oil flow visualization. For Reynolds numbers below 3 × 10⁶ the wedge boundary layer was first laminar, then transitional, and the corner interaction initially extended laterally well beyond the embedded shock. For Reynolds numbers greater than 3 × 10⁶ for which the boundary layer was turbulent, the interaction region barely extended beyond the embedded shock and the flow structure was found to be conically invariant and therefore essentially independent of viscous effects. Spanwise surface pressure distributions for both the laminar and the turbulent cases are virtually identical to those for two-dimensional flow separation.

Nomenclature

x = axial distance normal to the plane formed by the model leading edges (in.)
y = vertical distance normal to x, measured from the intersection of the wedges (in.)
z = horizontal distance normal to x, measured from the intersection of the wedges (in.)
p = pressure (psia)
p_t = Pitot pressure (psia)
M = Mach number
Re = local Reynolds number based on undisturbed wedge conditions

Subscripts

w = two-dimensional wedge conditions
∞ = freestream conditions
o = stagnation conditions

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Introduction

THE flow structure in a corner of intersecting wedges at supersonic speeds was first found experimentally by Charwat and Redekopp.[1] Their study revealed that the wedge bow shocks do not intersect, but rather, are joined by a third corner shock, and the internal flow includes two embedded shocks which terminate at the wedge surfaces, and a triangular region bounded by two slip surfaces and the corner shock. Subsequent studies with intersecting wedges[2,3] and flat plates[4] in hypersonic streams showed a similar corner flow structure.

Aside from a few turbulent surface heat transfer measurements by Stainbach and Weinstein[5] all experimental studies of corner interaction in supersonic and hypersonic flow were carried out at relatively low Reynolds numbers for which boundary layers were laminar. Thus, displacement effects, particularly in the forward region of the model, as well as viscous-inviscid interactions could be expected to influence the entire inviscid flow structure far downstream. Furthermore, experiments[1,5] revealed a considerable spread of disturbed flow extending laterally from the corner intersection far beyond the location of the embedded shock. The